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ORDER LEVEL INVENTORY MODEL FOR PERISHABLE ITEMS WITH POWER DEMAND AND STOCK-DEPENDENT DEMAND UNDER VARIABLE COST FUNCTIONS

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ABSTRACT

In real practice, customers are influenced to purchase more items if there is adequate availability of the products. The present paper investigates an order level inventory model under two scenarios having power demand rate with inventory level dependent holding cost functions, and stock-dependent demand rate with time and inventory dependent holding cost functions. A constant rate of deterioration is considered into the two models. Shortages are not allowed in the present models. In each of the two sections, the optimum time, optimum order quantity and optimum average total cost are derived. The developed models are illustrated by two numerical examples and finally the sensitivity analysis for the optimal solutions towards the changes in the values of system parameters has been discussed.

KEYWORDS: *Inventory, Perishable, Power Demand, Stock-Dependent Demand and Cost Function*

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INTRODUCTION

Perishable items are very common issues in our daily life. There are some items like steel, hardware, toys, glassware etc for which the rate of deterioration or decay or damage or devaluation is so slow that there is no need of consideration it in the development of order level inventory system. But other few types of perishable items like food items, drugs, pharmaceuticals and radioactive substances have sufficient deterioration in nature so that these are harmful for use and consequently this loss must be taken into account when analysing the system. In view of this, many researchers like Ghare and Schrader [1], Covert and Philip [2], Deng et al [3], Cheng et al [4], Ghiani et al[5],Rajoria et al [6], Dr. Biswaranjan Mandal [7] etc developed inventory models in which perishable items have been considered.

In many real life situations, for some consumer goods like fruits, vegetables, donuts etc, the consumption rate is influenced by the stock level. Usually it is observed that a large pile of goods on shelf in a supermarket will influence the customer to purchase more and so the consumption rate may vary with the stock level. Levin et al [8] discussed that a large amount of products showed in a shop will cause the purchaser to buy more items. Observation of Silver and Peterson [9] was " sales at the retails level tend to be proportional to stock displayed and a large amount of items displayed in a large shop will cause customers to purchase more". Baker and Urban [10] developed an inventory model for "a power stock induced demand pattern". After that Alfares[11] developed an inventory model assuming " inventory policy for an item with an inventory dependent demand rate and a storage time dependent holding cost, the holding cost per unit items per unit time is assumed to be an inventory function of the time spent in storage". Later many researchers like Datta and Pal [12], Hwang and Hahn [13], Tripathi et al [14] established an EOQ model for inventory-induced demand rate with holding cost functions.

In general, researchers developed their models assuming constant holding cost. But few researchers like Ray and Chaudhuri [15], Giri et al [16] developed inventory models where holding cost is time dependent. Many other researchers like Mukherjee et al [17], Tripathi et al [18], Sing et al [19] etc. are noteworthy. In the present paper, holding cost is assumed as in the two different functional forms of demand rate (i) inventory and (ii) time and inventory dependent.

In view of the above sort of situations and facts, an effort has been made to develop an order level inventory system for constant perishable items assuming two types models (i) power dependent demand and holding cost and (ii) inventory dependent demand and time dependent holding cost function. Shortages are not allowed in the present models. In each of these sections, the optimum time, optimum ordering quantity and optimum average total cost are derived. The developed models are illustrated by two numerical examples and finally the sensitivity analysis for the optimal solutions towards the changes in the values of system parameters has been discussed.

ASSUMPTIONS AND NOMENCLATURES

Assumptions

The fundamental assumptions used in this paper are given as follows:

- Lead time is zero.
- There is no repair of deteriorated items occurring during the cycle.
- Deterioration occurs when the item is effectively in stock.
- The demand rate is power demand pattern for model -I and stock-dependent for model-II.
- Holding cost follows constant for model-I and time dependent for model-II.
- Shortages are not allowed.

Nomenclatures

The following nomenclatures are used in the proposed model:

- i(t) : On hand inventory at time t.
- D(t) : Demand rate $D(t) = \sim i \ \ , \sim > 0, 0 < \ \ < 1$ for model-I

And

$$D(t) = \sim + \ \ i, \sim > 0, 0 < \ \ < 1$$ for model-II

Where

- $\sim$ is constant annual demand rate and is demand elasticity parameter.
- Q: Ordering quantity.

- $\bar{\ }$: Constant rate of deterioration of an item, 0< $\bar{\ }$ <1.
- T: Replenishment time.
- k: The ordering cost per order during the cycle period.
- h: The holding cost per unit time.
- c: The deterioration cost per unit item.
- HC : The holding cost function during the cycle period where

$$HC = \left| \begin{array}{l} \int_0^T h.i(t)dt \\ \int_0^T h.t.i(t)dt \end{array} \right\rangle$$

- $T = T_1^*$: Optimal cycle time for the model-I
- $T = T_2^*$: Optimal cycle time for the model-II
- $Q = Q_1^*$: Optimal order quantity for the model-I
- $Q = Q_2^*$: Optimal order quantity for the model-II
- $TC = TC_1^*$: Optimal inventory cost/time for the model-I
- $TC = TC_2^*$: Optimal inventory cost/time for the model-II.

MATHEMATICAL MODEL AND SOLUTION

The present model starts with attaining maximum inventory level Q units at t = 0. There is no shortages considered here and inventory level is depleted due to both deterioration and demand. So the mathematical model is developed under two proposed models.

Model I: Inventory Model with Power Dependent Demand and Holding Cost Function:

The on-hand inventory i(t) follows the following differential equation

$$\frac{di(t)}{dt} + \bar{\ }\, i(t) = -\sim\{i(t)\} \quad ,0 \le t \le T \tag{1}$$

The initial conditions are

$$i(0) = Q_1 \text{ and } i(T) = 0 \tag{2}$$

The solution of the equation (1) using (2) is given by the following

$$i(t) = [\frac{\sim}{}\{e^{-(1- \ )(T-t)} - 1\}]^{\frac{1}{1-}} \text{ , } 0 \le t \le T \qquad (3)$$

Since $i(0) = Q_1$, we get the following expression of the on-hand inventory from the equations (3)

$$Q_1 = [\frac{\sim}{}\{e^{-(1- \ )T} - 1\}]^{\frac{1}{1-}} \qquad (4)$$

The cost due to deterioration of units in the period [0,T] is given by

$$C_D = c^{-}\int_0^T i(t)dt = c^{-}\int_0^T [\frac{\sim}{}\{e^{-(1- \ )(T-t)} - 1\}]^{\frac{1}{1-}} dt$$

Since $0 < ^{-} < 1$ and $0 < 1- \ < 1$, neglecting higher order powers of $^{-}(1- \ )(<< 1)$ we get the following expression

$$C_D = c^{-}\frac{1-}{2-}\{\sim(1- \ )\}^{\frac{1}{1-}} T^{\frac{2-}{1-}} \qquad (5)$$

The inventory holding cost during the interval [0,T] is given by

$$C_H = h\int_0^T i(t)dt = h\int_0^T [\frac{\sim}{}\{e^{-(1- \ )(T-t)} - 1\}]^{\frac{1}{1-}} dt$$

Similarly neglecting the higher power terms of $^{-}(1- \ )(<< 1)$, we get the following

$$C_H = h\frac{1-}{2-}\{\sim(1- \ )\}^{\frac{1}{1-}} T^{\frac{2-}{1-}} \qquad (6)$$

Therefore the average total cost per unit time is given by

$$TC_1(T) = \frac{k}{T} + \frac{1}{T}C_D + \frac{1}{T}C_H$$

Substituting the values of C_D and C_H from the expressions (5) and (6), we get the following

$$TC_1(T) = \frac{k}{T} + (c^{-} + h)\frac{1-}{2-}\{\sim(1- \ )\}^{\frac{1}{1-}} T^{\frac{1}{1-}} \qquad (7)$$

For minimum, the necessary condition is $\frac{dTC_1(T)}{dT} = 0$

This gives $-\frac{k}{T^{2}}+(c^{-}+h)\frac{1-}{2-}\{\sim(1- \quad)\}^{\frac{1}{1-}}\frac{1}{1-}T^{\overline{1-}}=0$

Or, $T=\{\frac{k(2- \quad)}{c^{-}+h}\}^{\frac{1-}{2-}}\{\sim(1- \quad)\}^{\frac{1}{-2}}$ (8)

which gives the optimal cycle time $T=T_{1}^{*}$ for the model-I.

For minimum, the sufficient condition $\frac{d^{2}TC_{1}(T)}{dT^{2}}>0$ would be satisfied.

The minimum value of the average total cost of $TC_{1}(T)$ is TC_{1}^{*} and optimal order quantity Q is Q_{1}^{*} for the present model.

Model II: Inventory Model with Inventory Dependent Demand and Time Dependent Holding Cost Function:

For this case, the on-hand inventory i(t) follows the following differential equation

$$\frac{di(t)}{dt}+^{-}i(t)=-\{\sim+ \quad i(t)\},0\leq t\leq T \tag{9}$$

The initial conditions are

$$i(0)=Q_{2} \text{ and } i(T)=0 \tag{10}$$

The solution of the equation (9) using (10) is given by the following

$$i(t)=\frac{\sim}{^{-}+}\{e^{(^{-}+ \quad)(T-t)}-1\},\ 0\leq t\leq T \tag{11}$$

Since $i(0)=Q_{2}$, we get the following expression of the ordering quantity from the equations (11)

$$Q_{2}=\frac{\sim}{^{-}+}\{e^{(^{-}+ \quad)T}-1\} \tag{12}$$

From the expression (12), we get

$$T(=T_{2})=\frac{1}{^{-}+}\log(1+\frac{^{-}+}{\sim}Q) \tag{13}$$

The cost due to deterioration of units in the period [0,T] is given by

$$C_{D}=c^{-}\int_{0}^{T}i(t)dt=c^{-}\int_{0}^{T}\frac{\sim}{^{-}+}\{e^{(^{-}+ \quad)(T-t)}-1\}dt$$

Now integrating and then putting the value of T, we get the following expression by neglecting higher order powers of $(\bar{\ } + \ \ )T(<<1)$ as $0 < \bar{\ } < 1$ and $0 < \ \ < 1$,

$$C_D = \frac{c\bar{\ }}{2}\{Q + \frac{\bar{\ } + \ \ }{2\sim}Q^2\} \tag{14}$$

The inventory holding cost during the interval [0,T] is given by

$$C_H = h\int_0^T t.i(t)dt = h\int_0^T t.\frac{\sim}{\ \ + \ \ }\{e^{(\bar{\ } + \ \ )(T-t)} - 1\}dt$$

Similarly we get the following expression by neglecting higher order powers of $(\bar{\ } + \ \ )T(<<1)$,

$$C_H = \frac{h}{2\sim}Q^2 \tag{15}$$

Therefore the average total cost per unit time is given by

$$TC_2(T) = \frac{k}{T} + \frac{1}{T}C_D + \frac{1}{T}C_H$$

Substituting the values of C_D and C_H from the expressions (14) and (15), we get the following

$$TC_2(Q) = k\sim\{\frac{1}{Q} + \frac{\bar{\ } + \ \ }{2\sim} + \frac{(\bar{\ } + \ \ )^2}{4\sim^2}Q\} + \frac{c\bar{\ }}{2}(Q + \frac{\bar{\ } + \ \ }{2\sim}Q^2) + \frac{h}{2\sim}Q^2 \tag{16}$$

For minimum, the necessary condition is $\frac{dTC_2(Q)}{dQ} = 0$

This gives

$$k\sim\{-\frac{1}{Q^2} + \frac{(\bar{\ } + \ \ )^2}{4\sim^2}\} + \frac{c\bar{\ }}{2}(1 + \frac{\bar{\ } + \ \ }{\sim}Q) + \frac{h}{\sim}Q = 0 \tag{17}$$

which gives the optimal order quantity $Q = Q_2^*$ for the model-II.

For minimum, the sufficient condition $\frac{d^2TC_2(Q)}{dQ^2} > 0$ would be satisfied.

Putting the value of $Q = Q_2^*$ in the expressions (13) and (16), we get the optimal cycle time $T = T_2^*$ and the optimum average total cost $TC_2(T) = TC_2^*$.

A Particular Case

(a). Absence of deterioration (The deterioration of items is switched off i.e. $\bar{\ } = 0$)

Model I: Inventory Model with Power Dependent Demand and Holding Cost Function

The expressions (4) and (7) of order quantity (Q_1) and average total cost per unit time (TC_1) during the period [0,T] become

$$Q_1 = \{\sim(1- \quad)T\}^{\frac{1}{1-}} \tag{18}$$

And

$$TC_1(T) = \frac{k}{T} + h\frac{1-}{2-}\{\sim(1- \quad)\}^{\frac{1}{1-}} T^{\frac{1}{1-}} \tag{19}$$

The equation (3.8) becomes

$$T = \{\frac{k(2- \quad)}{h}\}^{\frac{1-}{2-}} \{\sim(1- \quad)\}^{\frac{1}{\quad -2}} = 0 \tag{20}$$

which gives the optimal cycle time $T = T_1^*$ for the model-I.

Model II: Inventory Model with Inventory Dependent Demand and Time Dependent Holding Cost Function:

The expressions (12) and (16) of order quantity (Q_2) and average total cost per unit time (TC_2) during the period [0,T] become

$$Q_2 = \frac{\sim}{\quad}(e^{\quad T} - 1) \tag{21}$$

And

$$TC_2(Q) = k\sim(\frac{1}{Q} + \frac{\quad}{2\sim} + \frac{\quad^2}{4\sim^2}Q) + \frac{h}{2\sim}Q^2 \tag{22}$$

The equation (17) becomes

$$k\sim(-\frac{1}{Q^2} + \frac{\quad^2}{4\sim^2}) + \frac{h}{\sim}Q = 0 \tag{23}$$

which gives the optimal order quantity $Q = Q_2^*$ for the model-II.

Numerical Examples

To illustrate the Model-I and Model-II, the following examples are considered:

Example 1: The values of inventory system parameter are $\sim$ = 6000 units/year; k = $500/order;

= 0.1; $\bar{\quad}$ = 0.2; c= $ 8 per unit; h= $ 10 per unit.

From the expressions (8), (4) and (7), we find the following optimum outputs for model-I

$T_1^* = 0.087$year; $Q_1^* = 944.50$ units and $TC_1^* =$ Rs.10860.70

It is checked that this solution satisfies the sufficient condition for optimality.

Example 2: The values of inventory system parameter are ~ = 6000 units/year; k = \$500/order;

= 0.1; ¯ = 0.2; c= \$ 8 per unit; h= \$ 10 per unit.

From the expressions (3.13), (3.12) and (3.16), we find the following optimum outputs for model-II

$T_2^* = 0.174$ year; $Q_2^* = 1068.83$ units and $TC_2^* =$ Rs. 4711.06

It is also checked that this solution satisfies the sufficient condition for optimality

SENSITIVITY ANALYSIS AND DISCUSSION

We now study the effects of changes in the system parameters ~ , k, , ¯ , c and hone the optimal cycle time (T^*), the optimal ordering quantity (Q^*) and the optimal average total cost (TC^*) in the present inventory models. The sensitivity analysis is performed by changing each of the parameters by – 50 %, – 20 %, +20% and +50 %, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in Table 1 and Table 2.

The following inferences can be made from the results in Table 1 and Table 2:

- In model-I and model-II, he optimal average total cost (TC^*) increase or decrease with the increase or decrease in the values of the system parameters ~ , k, , ¯ , c and h. The results obtained in model-I show that TC^* is highly sensitive to changes in the values of parameters ~ , k, and h.; and less sensitive to the changes of parameters ¯ and c. Whereas in model-II, it is observed that TC^* is highly sensitive to changes in the values of parameters ~ and k; and less sensitive to the changes in the values of parameters , ¯ , c and h.

- In model-I, the optimal ordering quantity (Q^*) increase or decrease with the increase or decrease in the values of the system parameters ~ , k and ; while Q^* increase or decrease with the decrease or increase in the values of the system parameters ¯ , c and h. On the other hand in model-II, the optimal ordering quantity (Q^*) increase or decrease with the increase or decrease in the values of the system parameters ~ and k; while Q^* increase or decrease with the decrease or increase in the values of the system parameters , ¯ , c and h. The results obtained in model-I show that TC^* is highly sensitive to changes in the values of parameters ~ , k, and h.; and less sensitive to the changes of in the values of parameters ¯ and c. Whereas in model-II, it is observed that Q^* is highly sensitive to changes in the values of parameters ~ , k and h; and less sensitive to the changes in the values of parameters , ¯ and c.

Table 1: Effect of Changes in the Parameters on the Model-I

Changing Parameter	% Change in the System Parameter	Optimum Values of		
		T_1^*	Q_1^*	TC_1^*
~	-50	0.126	658.45	7540.86
	-20	0.098	841.03	9657.23
	+20	0.079	1039.26	11954.52
	+50	0.071	1167.92	13444.27
k	-50	0.063	653.14	7540.86
	-20	0.079	839.13	9657.22
	+20	0.095	1040.92	11954.52
	+50	0.106	1171.44	13444.27
	-50	0.103	827.12	9494.99
	-20	0.093	894.72	10285.99
	+20	0.082	997.52	11477.11
	+50	0.074	1085.03	12487.85
–	-50	0.091	976.77	10499.22
	-20	0.089	956.46	10717.77
	+20	0.086	933.17	11001.60
	+50	0.085	916.15	11209.27
c	-50	0.091	981.21	10499.22
	-20	0.089	958.16	10717.94
	+20	0.086	931.56	11001.60
	+50	0.085	912.26	11209.27
h	-50	0.114	1274.86	8314.68
	-20	0.096	1044.59	9929.49
	+20	0.081	867.82	11710.63
	+50	0.074	780.90	12870.25

Table 2: Effect of Changes in the Parameters on the Model-II

Changing Parameter	% Change in the System Parameter	Optimum Values of		
		T_2^*	Q_2^*	TC_2^*
~	-50	0.222	689.34	3612.69
	-20	0.188	928.58	4322.18
	+20	0.162	1198.53	5059.14
	+50	0.149	1378.17	5523.07
k	-50	0.135	824.28	3086.57
	-20	0.160	983.67	4112.63
	+20	0.185	1143.40	5268.33
	+50	0.201	1241.19	6044.47
	-50	0.175	1069.96	4694.39
	-20	0.174	1069.31	4704.86
	+20	0.173	1068.45	4717.27
	+50	0.172	1067.81	4727.90
–	-50	0.186	1138.65	4228.38
	-20	0.178	1095.83	4521.37
	+20	0.169	1043.03	4897.82
	+50	0.162	1006.50	5171.34
c	-50	0.184	1137.15	4258.35
	-20	0.178	1095.01	4532.93
	+20	0.170	1044.06	4884.95
	+50	0.164	1009.35	5137.69
h	-50	0.203	1256.75	4156.92
	-20	0.183	1129.22	4509.72
	+20	0.166	1020.10	4893.17
	+50	0.157	961.59	5138.48

CONCLUDING REMARKS

In this paper, a perishable inventory model with power demand and stock-dependent demand rate under variable cost functions is developed in a planning cycle time. In model-I, holding cost is regarded as inventory dependent, where as in model-II, the holding cost function is considered as inventory dependent with time variable. This type of assumption is more realistic when the value of the unsold items decreases with time. Numerical examples are given to illustrate the models. Finally the sensitivity analysis of the system parameters is furnished. We may extend this model by assuming the demand as quadratic function of time, fully or partially backlogged shortages. Also the model may be improved by considering trade credit policy and inflation.

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